OpenFOAM capabilities for MHD simulation under nuclear fusion technology conditions

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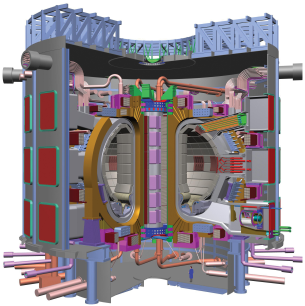
Outline

Nuclear Fusion Technology

ITER: International Thermonuclear Experimental Reactor

Demonstrate the scientific and technical feasibility of fusion power by 2016



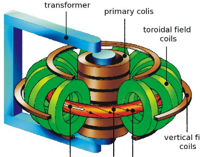
Main reaction:

$$^2_1H + \, ^3_1H \longrightarrow \, ^4_2He + \, ^1_0n_{(high\ e)}$$

at plasma's state (millions of ^oC)

Tokamak design:

torus-shaped fusion plasma confined by external magnetic field



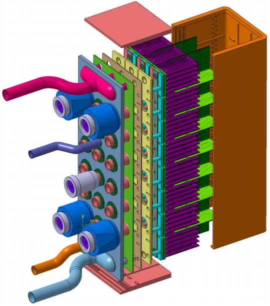
Critical aspects

1. very high plasma's temperatures (thousands of degrees)
   $\longrightarrow$ structural materials
   $\longrightarrow$ heat extraction system
2. plasma's confinement with powerful magnets under cryogenic conditions
   $\longrightarrow$ very high temperature gradient (materials?)
3. tritium as the main fuel. No natural tritium source.
   $\longrightarrow$ tritium generation
   $\longrightarrow$ tritium management and control (very high diffusivity)
4. radioactivity control (mainly for tritium)
   $\longrightarrow$ security factors (shielding of the components and environment)
   $\longrightarrow$ waste management

A key component: the Breeding Unit

Test Blanket Module

HCLL design for ITER



Pb as neutron multiplier

$^6_3Li + \, ^1_0n_{(low\ e)} \longrightarrow \, ^3_1H + \, ^4_2He$

- 3 main TBM functions:
  - Self-supplying of tritium
  - Heat power removing
  - Shielding against n and γ irradiation

$\longrightarrow$ need of detailed flow profiles

- Flow properties (HCLL):
  - $Ha = B\,L\,\sqrt{(\frac{\sigma_m}{\rho\nu})} > 10^4$
  - $N = \frac{Ha^2}{Re} \sim 10^3 - 10^5$
- Liquid metal (Pb-15.7Li)
  - $Rm = \sigma_m\,\mu_m\,v\,L << 1$

Research areas

1. Development of a MHD code
   - Low Rm numbers or full equations?
   - Reduction of CPU time: Wall functions
2. Coupling between MHD and heat transfer
   - Does Boussinesq hypothesis apply?
   - Dealing with very high source terms
   - High Ha effect on Rayleigh-Bénard instabilities
3. Tritium behavior in TBM
   - Treatment as a passive scalar: a post-process
   - Helium influence on tritium transport
   - Dealing with He bubbles

Outline

Physical background: governing equations

Assumed hypothesis:

- Incompressible fluid
- Constant fluid properties $(\rho, \mu, \sigma, \mu_m)$
- No body forces except Lorentz force (L)
- Grossly neutral fluid: $L = \rho_e \overrightarrow{E} + \overrightarrow{j} \times \overrightarrow{B} \rightarrow \overrightarrow{j} \times \overrightarrow{B}$

Governing equations

Navier-Stokes equations

$$\nabla \cdot \vec{v} = 0$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = -\frac{1}{\rho}\nabla p + \nabla \cdot (\nu \nabla \vec{v}) + \frac{1}{\rho}\left(\overrightarrow{j} \times \overrightarrow{B}\right)$$

Maxwell equations

Solenoidal nature of B: $\nabla \cdot \overrightarrow{B} = 0$

Faraday's law of induction: $\nabla \times \overrightarrow{E} = -\frac{\partial \overrightarrow{B}}{\partial t}$

Ampere's law: $\nabla \times \overrightarrow{B} = \mu_m \overrightarrow{j}$

Charge conservation: $\nabla \cdot \overrightarrow{j} = 0$

Ohm's law: $\overrightarrow{j} = \sigma_m(\overrightarrow{E} + \vec{v} \times \overrightarrow{B})$

Final set of equations

Bi-directional coupling

$$\nabla \cdot \vec{v} = 0$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = -\frac{1}{\rho}\nabla p + \nabla \cdot (\nu \nabla \vec{v}) - \frac{1}{2\rho\mu_m}\nabla \overrightarrow{B}^2 + \left(\frac{\overrightarrow{B}}{\rho\mu_m} \cdot \nabla\right)\overrightarrow{B}$$

$$\frac{\partial \overrightarrow{B}}{\partial t} + (\vec{v} \cdot \nabla)\overrightarrow{B} = (\overrightarrow{B} \cdot \nabla)\vec{v} + \nabla \cdot \left(\frac{1}{\sigma_m\mu_m}\nabla \overrightarrow{B}\right)$$

$$\nabla \cdot \overrightarrow{B} = 0$$

Applied at liquid metals...

Hypothesis: Low magnetic Reynolds Number ($Rm < 1$):

$$\vec{B} = \overrightarrow{Bo} + \vec{b} \sim \overrightarrow{Bo}$$

Uni-directional coupling

$$\nabla \cdot \vec{v} = 0$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = -\frac{1}{\rho}\nabla p + \nabla \cdot (\nu \nabla \vec{v}) + \frac{1}{\rho}\overrightarrow{j} \times \overrightarrow{Bo}$$

From the divergence of Ohm's law: $\nabla^2 \psi = \nabla \cdot (\vec{v} \times \overrightarrow{Bo})$

$$\overrightarrow{j} = \sigma_m (-\nabla \psi + \vec{v} \times \overrightarrow{Bo})$$

Algorithms

BPISO algorithm

Full coupling between velocity and magnetic field ($b \neq 0$)

$\nabla \cdot \overrightarrow{B} = 0 \quad \longrightarrow$ Projection Method from Brackbill & Barnes (1980).

$$\boxed{\begin{aligned} \overrightarrow{B}^* &= \nabla \times A + \nabla\psi \\ \Longrightarrow \quad \nabla \cdot \overrightarrow{B}^* &= \nabla^2\psi \\ \overrightarrow{B} &= \overrightarrow{B}^* - \nabla\psi \end{aligned}}$$

Conservative constraints

$\nabla^2\psi = \nabla \cdot (\nabla\psi)$ with:

1. the divergence scheme must be the same as the one used for $\nabla \cdot \overrightarrow{B}^*$ in the potential equation
2. the gradient scheme must be the same as the one used for the $\overrightarrow{B}$ correction

FSPM algorithm

Low magnetic Reynolds hypothesis ($b \sim 0$)

FSPM: Four Step Projection Method, from Ni et. al (2007)

A current density conservative scheme (*jn* is the current density at the cell faces)

$$L = \frac{1}{\rho}\overrightarrow{j} \times \overrightarrow{Bo}$$

$$\nabla^2\psi = \nabla \cdot (\vec{v_f} \times \overrightarrow{Bo_f})$$

$$\overrightarrow{jn} = \sigma_{m,f}\,(-\nabla_f\psi + \vec{v_f} \times \overrightarrow{Bo_f})$$

Conservative Constraints

$\nabla^2\psi = \nabla \cdot (\nabla\psi)$ with:

1. $\nabla\psi$ must be consistent in both Poisson equation and the *jn* evaluation
2. $\vec{v_f} \times \overrightarrow{Bo_f}$ must be consistent in both Poisson equation and the *jn* evaluation

Implementation

1. momentum equation
   $\{\partial U/\partial t+\nabla\cdot(\phi, U) = \nabla^2(\nu, U)-\nabla(B^2/(2\rho\mu_m))$
   $+\nabla\cdot(\phi_B/(\rho\mu_m), B)\} - (\nabla p)/\rho$
2. PISO loop
   1. Jacobi pre-conditioner for ϕ
      $U = H^u/D^u$
   2. Poisson equation for p
      $\nabla^2(1/(D^u\rho), p) = \nabla\cdot\phi$
   3. Correction $\quad U- = (\nabla p)/(D^u\rho)$
3. BISO loop
   1. magnetic field equation
      $\partial B/\partial t + \nabla\cdot(\phi, B) =$
      $\nabla\cdot(\phi_B, U) + \nabla^2(1/(\sigma_m\mu_m), B)$
   2. ϕ_B evaluation $\quad \phi_B = B_f\cdot S_f$
   3. Poisson equation for ψ
      $\nabla^2(1/D^B, \psi) = \nabla\cdot\phi_B$
   4. Correction $\quad B- = (\nabla\psi)/D^B$

1. FSPM loop
   1. momentum equation (Crank-Nicholson)
      $\{\partial U/\partial t+(\nabla\cdot(\phi, U) - \nabla^2(\nu, U)) - (j\times Bo)/\rho = 0\}$
   2. ϕ evaluation $\quad \phi = U_f\cdot S_f$
   3. Poisson equation for p
      $\nabla^2(1/(D^u\rho), p) = \nabla\cdot\phi$
   4. Correction $\quad U- = (\nabla p)/(D^u\rho)$
   5. χ evaluation
      $\chi = (\sigma_{m,f}(U_f\times B_f))\cdot S_f$
   6. Poisson equation for ψ
      $\nabla^2(\sigma_m, \psi) = \nabla\cdot\chi$
   7. jn evaluation
      $jn = -(\sigma_{m,f}(\nabla_f\psi\cdot S_f)) + \chi$
   8. j reconstruction

Stability and accuracy

Monotone scheme:

Constraint: all matrix components must be positive

Result: $\frac{|u|\Delta t}{\Delta x} + \frac{2\sigma_m\ B^2\Delta t}{\rho} \leq \frac{2\sigma_m\ B^2\Delta t}{\rho} + \frac{\nu\Delta t}{(\Delta x)^2} \leq 1$

Implemented: $\frac{|u|\Delta t}{\Delta x} + \frac{2\sigma_m\ B^2\Delta t}{\rho} + \frac{\nu\Delta t}{(\Delta x)^2} \leq 1$

Von Neumann analysis:

$U^{n+1} \ = \ A\ U^n$

Constraint: amplitude factor smaller than 1 for stability ($A \leq 1$)

Result: $L = \sigma_m\ B^2\ \Delta t\ /\ \rho \leq 2$

Best accuracy for $L \to 2$ or $L \to 0$

At $L = 2$ no phase error exists

Outline

Studied cases

Studied steady state cases with analytical solution:

1. Square channel with non conducting walls, 2D. **Shercliff (1953)**
2. Square channel with non conducting side walls and perfectly conducting Hartmann walls, 2D. **Hunt (1965)**

Boundary conditions

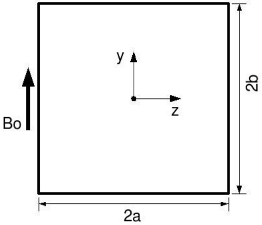
Non conducting walls:

$\rightarrow \quad B = Bo, \; b = 0 \qquad$ or $\qquad j = 0, \; \partial\psi/\partial n = 0$

Perfectly conducting walls:

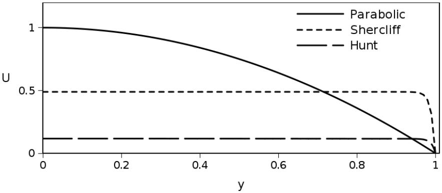
$\rightarrow \quad \partial B/\partial n = 0 \qquad$ or $\qquad \partial j/\partial n = 0, \; \psi = 0$

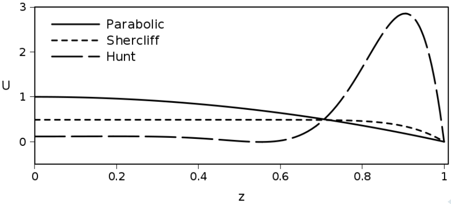
Flow profiles



1. Shercliff's case 1953: all walls perfectly insulating
2. Hunt's case 1965: conducting Ha walls

Example: Re=Ha=100





Set up

Time discretization:

- BPISO: backward Euler
- FSPM: backward Euler / Crank-Nicholson

Spatial discretization: Central Difference

Δt criterion: $\frac{|u|\Delta t}{\Delta x} + \frac{2\sigma_m\ B^2 \Delta t}{\rho} + \frac{\nu \Delta t}{(\Delta x)^2} \leq 1$

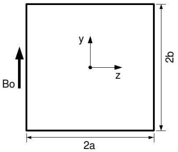
Steady state criterion: $\epsilon_u^n = \left| \frac{max\left(U^n - U^{n-1}\right)}{\lambda - 1} \right|$ where $\lambda = \frac{max\left(U^n - U^{n-1}\right)}{max\left(U^{n-1} - U^{n-2}\right)}$

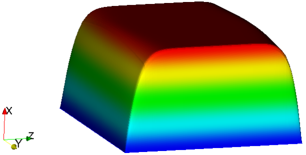
Solvers: (Bi-)Conjugate Gradient with incomplete-Cholesky pre-conditioner

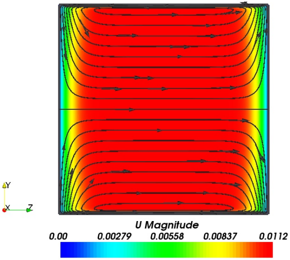
Shercliff's case

Adimensional numbers: $Ha = 10^2$, $Re = 10$, $N = 10^3$, $a = b = 1$
Imposed mass flow (periodic boundary conditions in x direction)

Electrically insulated walls

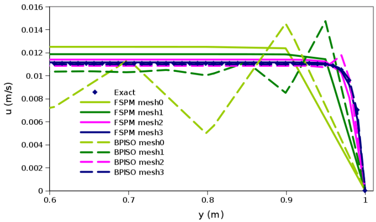


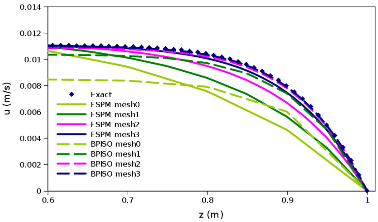




mesh	type	nodes	Ha nodes	side nodes
0	uniform	20×20	0.1	1
1	uniform	40×40	0.2	2
2	uniform	80×80	0.4	4
3	uniform	160×160	0.8	8
A	wall conc.	184×140	4	15

Shercliff's case: mesh errors





Algorithm	mesh	$max(\epsilon_y)\%$	$max(\epsilon_z)\%$	mean error %
FSPM	0	82.19	61.06	13.10
FSPM	1	67.12	45.39	8.21
FSPM	2	42.50	25.98	4.04
FSPM	3	11.06	9.50	1.45
FSPM	A	0.60	1.05	0.11
BPISO	0	78.96	49.39	91.53
BPISO	1	57.42	22.13	16.84
BPISO	2	30.59	8.19	2.91
BPISO	3	2.71	1.61	0.77
BPISO	A	1.11	0.63	0.15

Origin of the main errors:

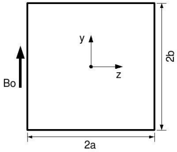
BPISO: behaves like a high order scheme, needs numerical diffusion

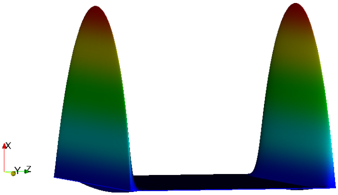
FSPM: in Ha boundary layers the diffusion term is not balanced by Lorentz forces unless the mesh is fine enough

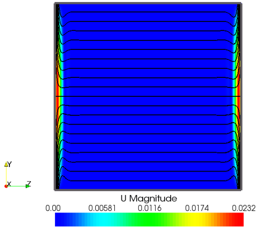
Hunt's case

Adimensional numbers: $Ha = 10^3$, $Re = 10$, $N = 10^5$, $a = b = 1$
Imposed mass flow (periodic boundary conditions in x direction)
Electrically insulated side walls

Conducting Ha walls

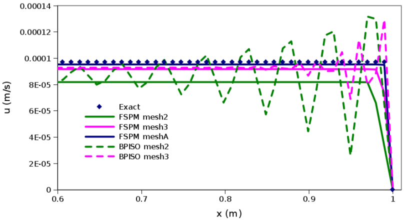


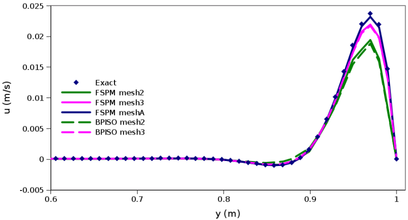




mesh	type	nodes	Ha nodes	side nodes
2	uniform	80×80	0.08	2.5
3	uniform	160×160	0.16	5.1
A	wall conc.	240×336	4	7.9

Hunt's case: mesh errors





Algorithm	mesh	$max(\epsilon_y)\%$	$max(\epsilon_z)\%$	mean error %
FSPM	2	65.98	60.27	17.26
FSPM	3	24.17	11.28	5.21
FSPM	A	2.20	11.43	1.70
BPISO	2	73.06	280.78	30.08
BPISO	3	33.18	56.94	7.67
BPISO	A	–	–	–

Outline

Conclusions

Main observations:

- BPISO behaves as a high order scheme, hence, presents oscillations near a discontinuity
- BPISO finds the steady state faster. Less CPU time for a coarse mesh
- BPISO presents more convergence problems at fine meshes. More CPU time for fine enough meshes
- FSPM is faster for each iteration

For an accurate result, we have chosen

the FSPM as the reference algorithm

Any question?

Thank you very much